# Effect of foundation flexibility on the elastic earthquake response of asymmetric structures

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ABSTRACT: A parametric study is presented of soil-structure interaction effects on torsional coupling in asymmetric buildings subjected to earthquakes. The study is based on a simple five-degree-of-freedom elastic interaction model consisting of a single-storey structural five-degree-of-freedom elastic interaction. The earthquake input is represented by idealisation with homogeneous half-space foundation. The earthquake input is represented by idealised design spectra, obtained from detailed studies of a large number of strong motion idealised design spectra. By comparing the lateral and torsional responses of identical records from the United States. By comparing the lateral and torsional responses of identical responds founded on both rigid and flexible foundations, the influence of soil-structure building models founded on both rigid and flexible foundations, the influence of soil-structure building models founded on both rigid and flexible foundations, on edge displacement. Use is interaction on torsional coupling is assessed for a range of building-foundation systems. Use is interaction on torsional comparisons are also made with the equivalent static design torque and shear of the building. Comparisons are also made with the equivalent static design torque and shear provisions of major building codes. The results indicate that for certain ranges of the provisions of major building codes. The results indicate that for certain ranges of the provisions of major building codes. The results indicate that for certain ranges of the provisions of major building codes. The results indicate that for certain ranges of the provisions of major building codes. The results indicate that for certain ranges of the provisions of major building codes. The results indicate that for certain ranges of the provisions of major building codes. The results indicate that for certain ranges of the provisions of major building codes. The results indicate that for certain ranges of the provisions of major building codes. The results i

#### 1 INTRODUCTION

Widespread investigations of linear and non-linear interaction effects on the response of structures have been presented in the literature (Whitman 1970, Liu & Fagel 1971, Chopra & Gutierrez 1974, Luco 1985), however for buildings which have structural eccentricities little is known of the effects of interaction on the resultant torsional response arising during earthquake vibrations. The Mexico City earthquake of 1985 resulted in the partial or total collapse of several buildings due to torsion (Chandler 1986), and as a result it has been suggested that the Mexican earthquake code design recommendations for torsional effects in asymmetric buildings (National University of Mexico 1977) do not include a sufficient safety margin.

The idealised structure—soil model employed in this study comprises a single—storey building with structural eccentricity perpendicular to the direction of earthquake input, resting on the surface of an elastic homogeneous half space (Figure 1). To facilitate analysis of response in the time domain, interaction forces at the building—soil interface are represented by frequency independent spring—dashpot

coefficients, making suitable adjustment to impedance values according to an approximate procedure which takes into account the influence of modal coupling and the values of the controlling system parameters. By varying parameters which represent dynamic soil properties, a wide range of foundation flexibility has been considered.

The building-foundation system in this form does not admit classical normal modes because of non-proportional damping in the foundation medium, and hence straightforward application of the response spectrum technique is not possible. The procedure proposed by Tsai (1974) and later developed for asymmetric buildings by Balendra, Tat & Lee (1982) is employed in this study to obtain approximate normal modes and the corresponding modal damping for the asymmetric shear building model. The technique matches rigorous and approximate normal mode solutions of the response of the structure for all five natural frequencies of the interaction system. The maximum structural response to earthquake input is then obtained by standard response spectrum techniques, employing the complete quadratic combination (CQC) method (Newmark & Rosenblueth 1971) to combine the modal maxima. A detailed parametric study has been

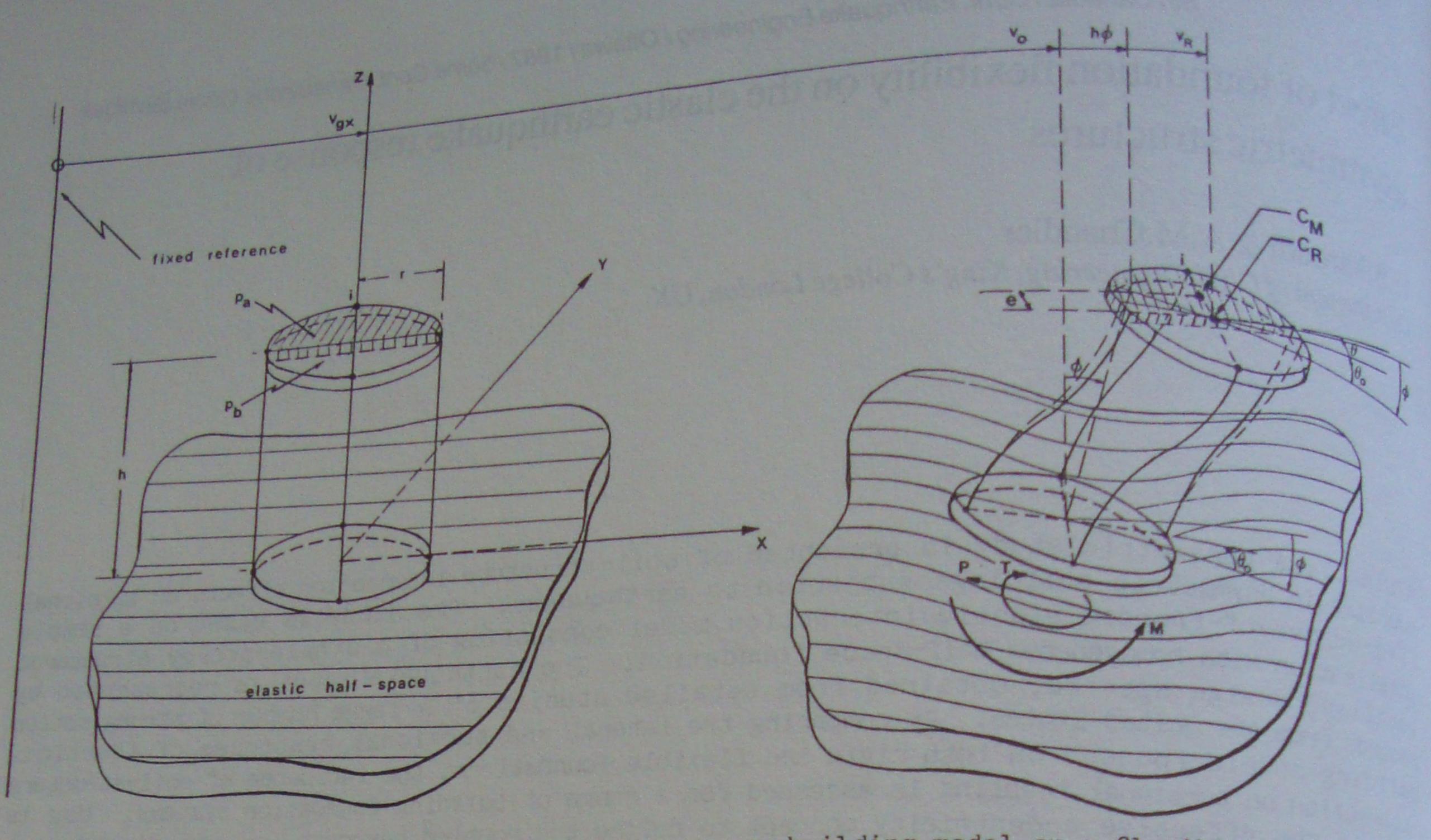


Fig.1. Response of a torsionally coupled single-storey building model on a flexible foundation

implemented for ranges of the key controlling parameters, typical of actual building-soil systems. First, analysis is made for a rigid foundation, comparing the dynamic floor lateral and torsional responses with the design provisions of major codes. The study is then extended to cover a range of flexible foundations representing typical soil properties, including suitable comparison with rigid foundation results in order to assess the importance of interaction on torsional response. Finally, comments are made on current design practice in the light of these results, and the need for revision in certain cases to allow for increased torsional response is indicated.

## 2 THEORETICAL DEVELOPMENT

# 2.1 Idealised single-storey interaction model

The structure-foundation model employed in the analysis is shown in Figure 1. A ris supported on an elastic homogeneous half-space of Poisson's ratio v, mass density diaphragm and foundation mat are idealised and have masses m and m, respectively. The floor rigid circular discs of negligible thickness static eccentricity e of the building, measured between the centre of mass CM and the with the centre of the floor disc), arises due pa and pb (pa)

pb) in the two halves of the disc, and is within the range  $0 \le e \le 0.424r$  (Chandler 1985). For a uni-directional horizontal component of free-field ground acceleration vgx, assumed to be uniform over the base of the building, the system has five degrees of freedom, viz: i) translation v<sub>M</sub> of the centre of mass CM in the x-direction due to structural deformation, ii) rotation  $\theta$  of the floor mass about the z-axis (through CR) due to structural deformation; and three degrees of freedom due to interaction at the building-soil interface, namely iii) translation vo of the foundation mass in the x-direction, iv) rotation  $\theta_o$  of the foundation mass about the z-axis, and v) rocking  $\phi$  of the whole building about the y-axis. Note that VM = VR - e0 (see Figure 1). The equations of motion for translation in the x-direction and rotation about CM are expressed as

$$\begin{bmatrix} \mathbf{m} & \mathbf{0} \\ \mathbf{0} & \mathbf{m} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{\mathsf{M}} \\ \mathbf{0} & \mathbf{t} \end{bmatrix} + \begin{bmatrix} \mathbf{c}_{\mathsf{V}} & \mathbf{e}_{\mathsf{C}_{\mathsf{V}}} \\ \mathbf{e}_{\mathsf{C}_{\mathsf{V}}} & \mathbf{c}_{\mathsf{\theta}\mathsf{M}} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{\mathsf{M}} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{\mathsf{V}} & \mathbf{e}_{\mathsf{K}_{\mathsf{V}}} \\ \mathbf{c}_{\mathsf{C}_{\mathsf{S}}} \end{bmatrix} \begin{bmatrix} \mathbf{K}_{\mathsf{V}} & \mathbf{e}_{\mathsf{K}_{\mathsf{V}}} \\ \mathbf{v}_{\mathsf{M}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{K}_{\mathsf{V}} & \mathbf{e}_{\mathsf{K}_{\mathsf{V}}} \\ \mathbf{e}_{\mathsf{K}_{\mathsf{V}}} & \mathbf{K}_{\mathsf{\theta}\mathsf{M}} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{K}_{\mathsf{V}} & \mathbf{e}_{\mathsf{K}_{\mathsf{V}}} \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{\mathsf{M}} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{K}_{\mathsf{V}} & \mathbf{K}_{\mathsf{\theta}\mathsf{M}} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{K}_{\mathsf{V}} & \mathbf{K}_{\mathsf{\theta}\mathsf{M}} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

where  $v_M$  is the total displacement of  $C_M$  in the x-direction (=  $v_{gx}$  +  $v_o$  +  $h\phi$  +  $v_M$ ),  $\theta^t$  is the total twist of the floor about the z-axis (=  $\theta_o$  +  $\theta$ ), l is the radius of gyration about  $C_M$  and the corresponding mass moment of

inertia JeM = ml2; Kv and KeM are the lateral inertia JeM stiffnesses defined at C inertia dem at iffnesses defined at CR and and torsional stiffnesses defined at CR and respectively, Cv and CAM and and torsioned ively, cy and command are the corresponding damping for the superstructure, correspond to be proportional to the stiffner. correspond to be proportional to the stiffness as defined by:

The constant of proportionality, B, is The ton the basis of five per cent of evaluated on the basis of five per cent of evaluate damping in the fundamental mode of critical damping (Balendra et al 1000) critical mode of the superstructure (Balendra et al 1982). The the super stiffness and corresponding damping torsional Stiffness and corresponding damping defined at CR are given by:

The equations of motion for the whole system are expressed as:

$$\frac{\ddot{v}}{(\dot{v} + \dot{v}_0)} + \frac{\ddot{w}}{v_0} + \frac{\ddot{v}}{v_0} + \frac{v}{v_0} + \frac{\ddot{v}}{v_0} + \frac{\ddot{v}}{v_0} + \frac{\ddot{v}}{v_0} + \frac{\ddot{v}}{v_$$

$$m_{0}(\ddot{v}gx + \dot{v}o) + 1^{2}m\ddot{\theta}t + T(t) = 0$$

$$1^{2}m_{0}\ddot{\theta}o + 1^{2}m\ddot{\theta}t + M(t) = 0$$
(4b)

are expression 
$$\frac{\ddot{v}_{0}}{m_{0}} = \frac{\ddot{v}_{0}}{v_{0}} + \frac{\ddot{v}_{0}}{v$$

where Iyj (j=0,1) are moments of inertia of the base and floor masses respectively about the y-axis, P(t), T(t) and M(t) are interaction forces for translation, rotation and rocking respectively (Figure 1). These interaction forces are approximated by frequency independent spring-dashpot coefficients (Richart, Hall & Woods 1970) in the form:

$$P(t) = C_T v_0 + K_T v_0$$
 (5a)

$$T(t) = CZ\dot{\theta}_0 + KZ\theta_0$$
 (5b)
$$T(t) = CZ\dot{\theta}_0 + KZ\theta_0$$
 (5c)

$$T(t) = CZ\theta_0 + KZ\phi_0$$

$$M(t) = CR\dot{\phi} + KR\phi$$
(5c)

where the coefficients CT, CZ, CR, KT, KZ, KR have been developed according to the method of Ghaffar-Zadeh and Chapel (1983).

## 2.2 Approximate normal mode method

Putting vMt = vM' + vgx (where vM' = vo + ho + VM), equations (1) and (4) can be written as

$$[M']{\{Y\}} + [C']{\{Y\}} + [K']{\{Y\}} = -{P'}$$
 (6a)

Where

$$[M'] = \begin{bmatrix} [M_S] \\ m_o \\ 1^2 m_o \\ Iy_o + Iy_1 \end{bmatrix}$$
 (6d)

$$[C'] = \begin{bmatrix} \begin{bmatrix} c_S \end{bmatrix}^{-c_V} & -c_V e & -c_V h \\ -c_V e & -c_{\theta M} & -ec_V h \\ c_T + c_V & ec_V & c_V h \\ c_Z + c_{\theta M} & ec_V h \\ c_R + h^2 c_V \end{bmatrix}$$
(6e)

and matrix [K'] is obtained by replacing [CS] with [KS]; cv and com with Kv and Kom; CT, CZ and  $C_R$  with  $K_T$ ,  $K_Z$  and  $K_R$  in equation (6e). Define the following transformation:

$${Y} = [A_1]{r}$$
 (7a)

in which 
$$[A_1] = \begin{bmatrix} [\Phi] \\ |1/(m_0)|/2 \\ |1/(I_{y_0} + I_{y_1})|/2 \end{bmatrix}$$
 (7b)

where 
$$[\Phi] = \begin{bmatrix} \phi_V^1 & \phi_V^2 \\ \phi_{\theta}^1 & \phi_{\theta}^2 \end{bmatrix}$$
 (7c)

is the [2x2] normal mode matrix for the superstructure, satisfying:

$$[\Phi]^{T} [M_{S}][\Phi] = [I] \qquad (8a)$$

$$[\Phi]^{T} [K_{S}][\Phi] = [\omega_{K}^{2}]$$
(8b)

$$[\Phi]^T [C_S][\Phi] = [2\zeta_{K}\omega_{K}]$$
(8c)

in which  $\omega_k$  (k=1,2) are the lateral and torsional circular frequencies and  $\zeta_k$  (k=1,2) are the modal fractions of critical damping for the superstructure  $(\zeta_1=0.05, \zeta_2=\beta\omega_2/2)$ , and [I] is a unit matrix. Premultiplying equation (6a) by [A,]T and applying equation (7a) gives:

$$\{\ddot{r}\} + [\hat{c}]\{\dot{r}\} + [\hat{K}]\{r\} = -\{\hat{\gamma}\}$$
 (9a)

where 
$$[\hat{K}] = \begin{bmatrix} [K_U] \mid [K_{UL}]^T \end{bmatrix}$$
 (9b)  $[K_{UL}] \mid [K_L] \mid [K_L] \end{bmatrix}$  5x5

$$[K_U] = [\omega_{k}^2]_{2X2}, k=1,2$$
 (9c)

and the submatrices [KUL] and [KL] are given by Balendra et al (1982). The matrix [C] is obtained by replacing [wk2] by [25kwk] and KT, Kz and KR by CT, Cz and CR in [K]. Furthermore

$$\{\hat{\gamma}\}T = \{m\phi_{V}^{1} m\phi_{V}^{2} (m_{o})^{1/2} \ddot{v}gx = 0 0\}$$
 (9d)

Consider a second coordinate transformation

$$\{r\} = [A_2]\{q\}$$
(10a)
$$\{r\} = [A_2]^T[A_2] = [I]$$
(10b)
$$\{n \text{ which } [A_2]^T[A_2] = [I]$$
(10c)

$$\{r\} = [A_2]\{q\}$$

$$\text{in which } [A_2]^T[A_2] = [I]$$

$$[A_2]^T[\hat{K}][A_2] = [\overline{\omega}_{K}^2], \text{ k=1 to 5 (10c)}$$

where  $\overline{\omega_k}$ , k=1 to 5, are the natural circular frequencies of the building-foundation system. Substituting equation (10a) into (9a), and premultiplying by [A2]T gives:

$$\{q\} + [C]\{q\} + [\overline{\omega}_{K}^{2}]\{q\} = -\{\overline{Y}\}\$$
 (11a)

where 
$$\begin{bmatrix} \overline{C} \end{bmatrix} = \begin{bmatrix} A_2 \end{bmatrix}^T \begin{bmatrix} \hat{C} \end{bmatrix} \begin{bmatrix} A_2 \end{bmatrix}$$
 (11b) (11c) (11c) (11c)

In equation (11a), the 5 equations are coupled by the off-diagonal terms in [C]; hence in applying the normal mode method it is assumed that equation (11a) can be approximated by the following uncoupled

equations: 
$$(12)$$

$$(\ddot{q}) + [2\zeta_{k}\overline{\omega}_{k}]{\dot{q}} + [\overline{\omega}_{k}^{2}]{q} = -{\Upsilon}$$

The response vector in equation (6a) can be

approximated to: (13)

approximated to:  

$$(\hat{Y}) = [\Delta]\{q\}, \text{ where } [\Delta] = [A_1][A_2]$$
 (13)

# 2.3 Estimation of modal damping

By assuming harmonic ground motion  $v_{gx}=e^{i\omega t}$ , where w is the circular frequency of the ground excitation, the exact solution for response may be expressed as:

$$\{Y(\omega)\}=\omega^{2}[A_{1}][\hat{K}]-\omega^{2}[I]+i\omega[\hat{C}]^{-1}\{\hat{Y}\} \qquad (14)$$

and the approximate normal mode solution is:

$$\{\hat{Y}(\omega)\} = \omega^2 [\Delta] [[\overline{\omega}_{\mathsf{K}}^2] - \omega^2 [I] + i\omega [2\overline{\zeta}_{\mathsf{K}} \overline{\omega}_{\mathsf{K}}]^{-1} \{Y\} \quad (15)$$

The modal damping Tk is then determined by matching the exact and approximate response from equations (14) and (15), at frequencies  $w = \overline{w}_{k}$  (k=1 to 5), and for a dominant approximate modal response component in each mode considered. For this study, these have been established as response components 1,2,4 for modes 1,2,4 and the maximum approximate response component for modes 3 and 5. Hence

$$|Y_{j}(\overline{\omega}_{k})| = |\hat{Y}_{j}(\overline{\omega}_{k})|, k=1 \text{ to } 5$$
 (16)

and j is the degree of freedom as given above. A first estimate of  $\zeta_k$  is obtained by neglecting the off-diagonal terms in [C] (equation (11a)), hence

$$\overline{\zeta}_{K} = \frac{\overline{C}_{KK}}{2\overline{\omega}_{K}}, \quad k=1 \text{ to 5}$$

$$RV \quad V = 2\overline{\omega}_{K} \quad (17)$$

By varying the damping ratio corresponding to mode 1  $(\zeta_1)$  whilst keeping the others unaltered, the right-hand side of equation (16) is iterated until the responses are equal. The second damping ratio  $\overline{\zeta}_2$  is then evaluated using the established value for  $\zeta_1$  and values given by equation (17) for the remainder. Similarly,  $\overline{\zeta}_3$ ,  $\overline{\zeta}_4$  and  $\overline{\zeta}_5$  are established.

# 2.4 Earthquake spectra

The earthquake input used in this study is represented by design spectra as proposed by Hall, Mohraz & Newmark (1976), obtained from a statistical study of 85 accelerogram records

from western United States spanning period beginning 1933. Based on a 39 year maximum ground motion and a na a great mulative probability of exceedance, specified tion factors for displacetors. maximum ground in aximum ground ground in aximum ground groun replification ration are specified velocity and acceleration are specified to Hence, the result across different critical damping ratios and across and across tripartite logarity could frequency range. Hence, the results across and across could be set used in the results could be presented in a tripartite logarithmic could a presented in set used in this study resented in a traper used in this study in this study is ratio (v = maximum ground acceleration) he ground motion and maximum ground acceleration of 0.3g, a (v/a) ratio (v = maximum ground acceleration) a, of 0.3g, a (v/a) ratio (v = maximum ground at iffness) atiffness) velocity) of 48 in/sec/g (corresponding ground mediate soil stiffness), and an add intermediate soil stiffness), and an adding to a addin ratio (d = maximum ground displacement) ad/with represents to ensure that the spectrum represents of the sp adequate band (frequency) width to accomodate adequate. Also a cumulate a range of earthquakes. Also a cumulative probability of exceedance of 15.8% (median plus one standard deviation) is chosen, which is a value commonly used in design spectra to incorporate an adequate degree of

Figure 2 shows acceleration spectra normalised to a maximum ground acceleration of 0.3g, vs natural period for damping of 2,5, 10, 20, 30 and 40 per cent of critical. Also shown on the figure are normalised 5% acceleration spectra for the El Centro earthquake of 1940 and Romanian earthquake of 1977. The design spectrum for 5% damping provides a reasonable representation of the corresponding El Centro spectrum throughout the period range shown; in contrast the same design spectrum gives a poor match with the Romanian curve, particularly for periods longer than 1 sec where there is a significant underestimation of the calculated response, This highlights the differences in spectral shape observed in earthquakes recorded under different localised conditions, which will be discussed with reference to torsional response effects in section 3.3.

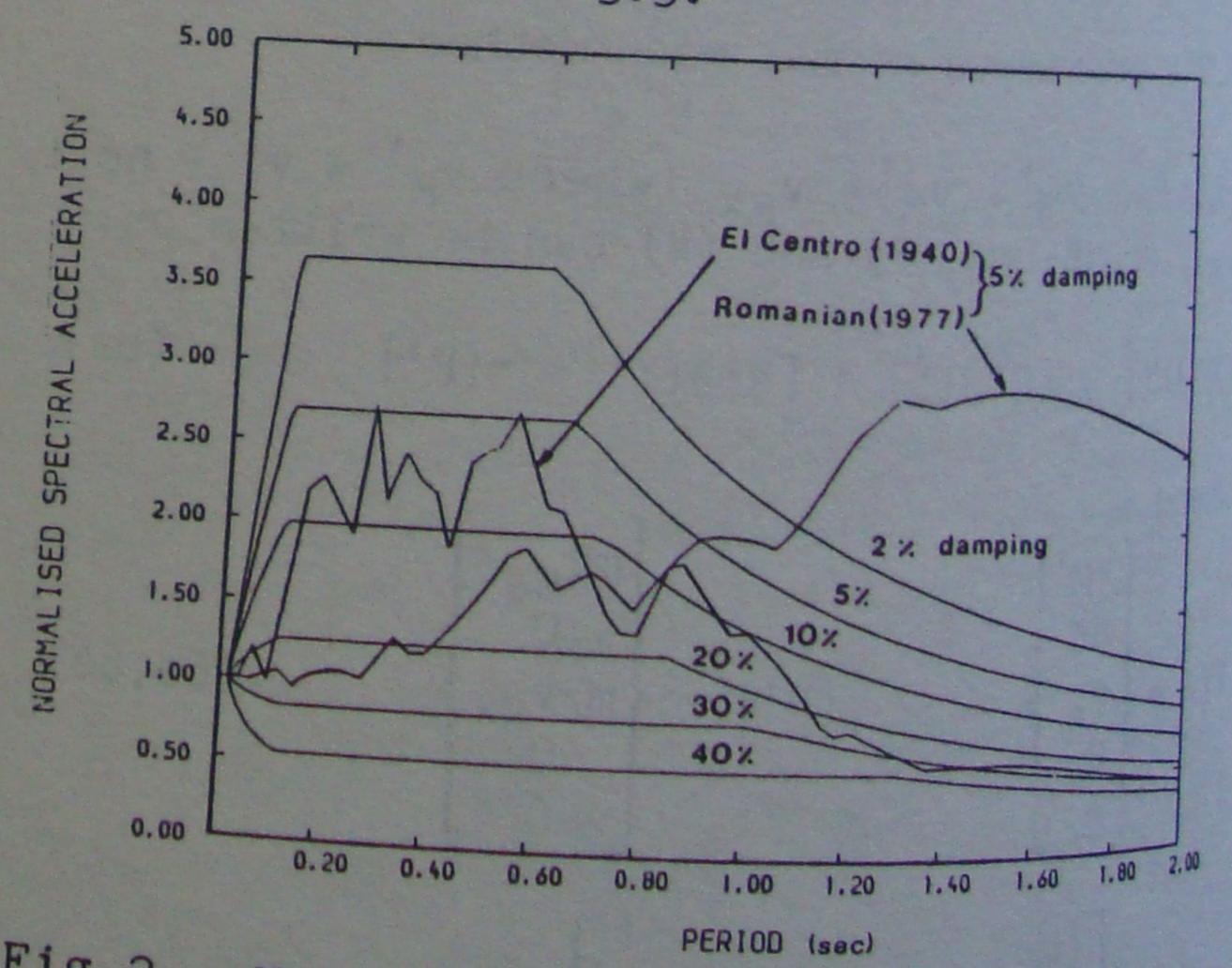


Fig. 2. Normalised design acceleration spectra for various damping values (after Hall, Mohraz and Newmark 1976), together with response Spectra for 5 per cent damping for the El Centro (1940) and Romanian (1977) earthquakes

3 PARAMETRIC RESPONSE OF RIGIDLY BASED STRUCTURE

3.1 Choice of parameters

In order to carry out a detailed study of the In order the system, the parameters influencing the response are developed in a influensional form and implemented in the non-unitions of motion (1) and (4). The parameters employed are listed in Table 1, and parameter of the non-dimensional terms  $e_r$ ,  $\lambda_T$ ,  $\alpha$  consist of the following values is and  $\delta h^i$  to parameters whose income and off to parameters whose influence on the trend of response has been found to be negligible: v=1/3,  $\delta_{m}=0.3$ ,  $\delta_{p}=0.14$ . The natural frequencies of the system are calculated on the basis of the uncoupled translational natural period Ty, relating to the rigidly based building. Parameter  $\alpha$  in Table 1 is a measure of the structurefoundation stiffness and is a function of the total height H of the building (see below), the shear wave velocity Vs of the foundation medium and the fundamental coupled frequency w, of the rigidly based structure. Values of  $\alpha$  considered in this paper are  $\alpha=3$ , 6, 10 and . the first three values corresponding approximately to shear wave velocities of 100, 200 and 330 m/s, respectively, and the latter value corresponding to a rigid foundation. Although some differences in results are observed between  $\alpha=10$  and  $\alpha=\infty$ , these are not large; consequently values of a intermediate between 10 and infinity are not presented. The parameters er and AT are of special significance in studies of torsional coupling in rigidly based buildings (Chandler & Hutchinson 1987); er is a measure of the static eccentricity e/r of the structure and AT is the ratio of uncoupled torsional and translational natural frequencies  $\omega_{A}/\omega_{V}$ , where

$$\omega_{V}=2\pi/T_{V}=(K_{V}/m)^{1/2}; \omega_{\theta}=(K_{\theta}R/J_{\theta}R)^{1/2}$$
 (18)

The terms  $K_V$ ,  $K_{\theta R}$  and  $J_{\theta R}$  (=  $J_{\theta M}$ -me<sup>2</sup>) have been defined in section 2.1.

By assuming an empirical relationship  $T_v=0.1N$  (Applied Technology Council 1978) between the natural period  $T_v$  and N, the number of building storeys, the following relationships are derived:

$$m = 10T_{vm_S}$$
  
 $h = (H+h_S)/2$  (H=Nh<sub>S</sub>) (19a)  
(19b)

where m<sub>s</sub> is the floor mass of each storey (assuming a floor intensity of 5.2 kN/m<sup>2</sup> and r=10m), h<sub>s</sub> is the inter-storey height (taken as 3.75m), and H is the total height of the building. The building mass m, together with height h measured to the overall centre of mass of the N-storey building, are employed in this study with the single-storey idealisation shown in Figure 1.

### 3.2 Torsional code provisions

The design storey torque given by building codes comprises two parts:

$$T_b = T_{eb} + T_a \tag{20}$$

where  $T_{eb}$  and  $T_a$  are equivalent static torsional moments accounting for static and accidental eccentricity respectively. The latter term accounts, approximately, for a number of effects which cannot be quantitatively determined (Chandler 1985). The components  $T_{eb}$  and  $T_a$  are written in general form

$$T_{eb} = aeQ_{vu}$$
 (21a)  
 $T_{a} = bYQ_{vu}$  (21b)

where Y is the plan dimension of the building perpendicular to the earthquake direction,  $Q_{yu}$  is the uncoupled (design) shear corresponding to e=0, and a,b are coefficients whose values

Table 1. Dimensionless parameters for single-storey torsionally coupled interacting systems.

Гуре	Description	Definition	Symbol
Structural parameters	Static eccentricity ratio Torsional to translational freq. ratio Mass ratio Height ratio	e/r ω <sub>θ</sub> /ω <sub>ν</sub> m <sub>o</sub> /m h/r	er λT δm δh
Soil	Poisson's ratio		V
Interaction parameters	Density ratio Shear wave velocity ratio	$m/\pi r^2 hp$ $V_S/H(\omega_1/2\pi)$	δρα

for the codes considered are specified in Table 2. The design eccentricity is given by eb = Tb/Qvu = ae + bY, and the dynamic eccentricity ratio is defined as:

$$(22)$$

 $e_{dr} = e_{d}/r = T_{eb}/rQ_{vu} = ae_{r}$ 

where the coefficient, a, represents the code allowance for torsional coupling, in terms of the amplification of static eccentricity required to induce the design torsional moment Teb.

Table 2. Code coefficients for design eccentricity.

		b
Building codes	a	
ATC3 (California) Canada Mexico New Zealand Eurocode No.8	1.0 1.5 1.5 1.7-0.5er 1+e,r/er	0.05 0.10 0.10 0.10 0.05

The supplementary eccentricity ratio eir specified by Eurocode 8 (1984) is assigned the smaller of the two values computed from the following expressions:

$$e_{1}r=0.894(e_{r})^{\frac{1}{2}}, \leq 0.4$$
 (23a)

or 
$$e_{1}r=(0.5/e_{r})\{0.667-e_{r}^{2}-0.5\lambda_{T}^{2}+\frac{1}{2}\}$$
 (23b)

Furthermore, if  $\lambda T^2 > 10(0.667 + ep^2)$ , then eir is taken to be zero.

## 3.3 Effective eccentricity approach

The effective eccentricity ee(i) (Dempsey & Tso 1982) represents the primary design requirement (that is, for the displacement of edge element i in Figure 1), and is found by matching the peak displacement vi max of element i as obtained by dynamic analysis with that corresponding to an effective storey torque given by  $T_{eb} = e_e(i)Q_{vu} = e_{er}(i)(rQ_{vu})$ . Qvu represents the design storey shear as specified by all the building codes in section 3.2, determined by analysis of the response of the uncoupled structure to specified earthquake loading. Hence  $Q_{vu} = K_v v_u(\omega_v, \zeta)$ where vu is the peak (spectral) displacement. The expression for primary design effective eccentricity ratio is then given by (Chandler

$$e_{er}(i) = \lambda T^{2}(R_{i}-1)/2$$
here R: (2011)

where Ri is the coupled to uncoupled edge

The variation of eer(i) with er for rigidly The variation in Figures 3 (a) rigidly based buildings is shown in Figures 3(a) rigidly based buildings is shown the maximum effect. based buildings to shows the maximum of a)-(c). Figure 3(a) shows the maximum effective Figure 3(a) since figure 3(a) since obtained from dynamic eccentricity envelopes obtained from dynamic eccentricity enveloped a spectra illustrated in response to the design spectra illustrated in response for Ty of 0.1, 0.2, 0.5 and 2.0 and 2.0 response to the response to the of 0.1, 0.2, 0.5 and 2.0 sed in Figure 2, for Ty of critical damping in the second Figure 2, for TV and assuming 5% of critical damping in the antal mode. For each value as and assuming of the proportion of the proportion fundamental mode of the proportions (corresponding to buildings of the proportions (corresponding to buildings of the proportions) (corresponding 3(a)), the envelopes have been shown in Figure 3(a)), the envelopes have been shown in Figure 5 been drawn from results obtained over a range of to the drawn of and 2.0, corresponding to between 0.6 and 2.0, corresponding to most between 0.0 and (Hart, DiJulio & Lew 1975), actual buildings (Hart, DiJulio & Lew 1975). shown on the same figure are the average shown on the curves for a range of Ty

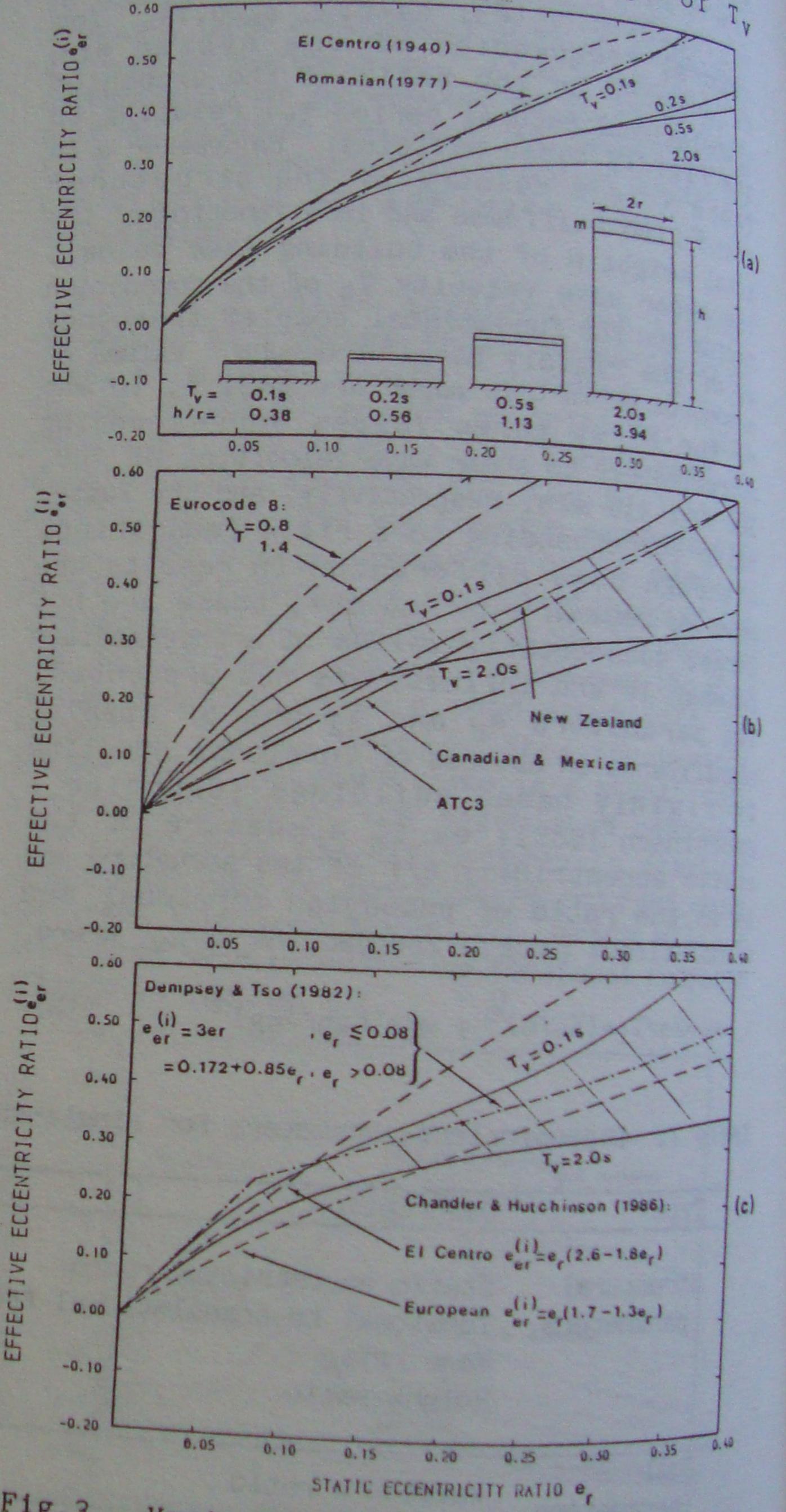


Fig. 3. Variation of envelopes of effective eccentricity ratio with er for rigidly based buildings, and comparison with response to the El Centro and Romanian earthquakes (a), with the dynamic eccentricity provisions of major building codes (b), and with various alternative design proposals (c)

corresponding to the El Centro and Romanian corresponding to the El Centro and Romanian correspond obtained by time history analysis earthquakes, damping in both vibration earthquakes, damping in both vibration modes.

assuming 5% obtained using design assuming of the state of the assuming obtained using design spectra the results torsional coupling is especial The result torsional coupling is especially indicate that for short-period building indicate for short-period buildings, as significant view of the spectral shape significant view of the spectral shape. For expected in view history curves mate expected the time history curves match the er alts obtained using design spectrum. er obtained using design spectra; for results obtained er, time history and spectra time history results values of er, time history analysis higher the more conservative of nigher the more conservative effective yields the requirements for the yields requirements for the records eccentricity requirements for the records eccentric Despite the variation in spectral chosen. Despite El Centro and Despite the El Centro chosen. the El Centro and Romanian shape between the El Centro and Romanian shape of the shape of the effective earthquakes (Figure 2), the effective earthque envelope averaged over a range of eccentricity envelope averaged over a range of eccentrice of and 2.0 sec) is relatively between 0.1 and 2.0 sec) is relatively consistent (Figure 3(a)).

Figure 3(b) compares the results of this study for  $T_V=0.1$  and 2.0 sec (the upper and study bounds to the results shown in Figure 3(a)) with the corresponding building code provisions for dynamic eccentricity ratio (equation (22), see also Table 2). The results of earlier studies (Tsichias & Hutchinson 1981, Chandler & Hutchinson 1987) are confirmed; namely, the ATC3 (Applied Technology Council 1978), Canadian (National Building Code of Canada 1985), Mexican (National University of Mexico 1977) and New Zealand (Standards Association of New Zealand 1976) codes are non-conservative to varying degrees for buildings with small eccentricity (er (0.15). For higher eccentricity ratios, the Canadian, Mexican and New Zealand codes give reasonable estimates of response, whilst the ATC3 code remains non-conservative to a decreasing extent. The Eurocode 8 (1984) provisions, shown for \pmov1=0.8 and 1.4, are conservative throughout and especially for large eccentricities.

A further assessment of dynamic results is made in Figure 3(c), in comparison with the bi-linear and parabolic approximations of effective eccentricity suggested by Dempsey & Tso (1982) and Chandler & Hutchinson (1986), respectively. In the latter case two curves are shown corresponding to the computed responses to the El Centro (1940) earthquake, and the average response to a series of 7 recent strong-motion European earthquakes including the Romanian earthquake of 1977. The Dempsey and Tso envelope is reasonable for er (0.10, but is non-conservative compared with the higher of the dynamic response envelopes (Ty = 0.1 sec) for er > 0.15. The average results for European earthquakes indicate less torsional coupling for Structures with small eccentricities (er 0.2) than obtained using the design spectra of Figure 2.

- 4 EFFECT OF INTERACTION ON LATERAL-TORSIONAL COUPLING
- 4.1 Dynamic amplification of shear and torque

To understand torsional coupling effects in elastically founded buildings by a study of parametric response trends (enabling comparisons with building code provisions), it is useful to consider these effects in relation to the key structural parameters er,  $\lambda_T$  and the foundation stiffness parameter  $\alpha$ , by averaging selected response quantities over a wide range of  $T_V$ . The trends observed are then representative of buildings with a range of natural periods and their general applicability is enhanced. These characteristics are examined using two response ratios:

- (i) shear amplification ratio  $(Q_V/Q_{VU})$ , where  $Q_V$  is the dynamic storey shear calculated for the coupled system. Both  $Q_V$  and  $Q_{VU}$  are calculated for the interaction model, the uncoupled structure having three degrees of freedom.
- (ii) amplification of eccentricity (e<sub>dr</sub>/e<sub>r</sub>), where e<sub>dr</sub> is given in equation (22), T<sub>eb</sub> being the dynamic storey torque calculated for the coupled system.

Figure 4(a) illustrates the variation of the shear amplification ratio Qv/Qvu with \lambda\_T, for models with  $\alpha=3$ , 6, 10 and  $\infty$  (a range from very soft to infinitely rigid soils). The structure-soil model is subjected to free-field earthquake spectra as shown in Figure 2, taking damping in the fundamental mode to be 51=0.05 and eccentricity ratio er=0.3. Furthermore the results presented are based on the average response of systems with Ty in the range 0.1-2.0 sec. The recommendation of building codes,  $Q_v/Q_{vu} = 1$ (i.e. no allowance for shear reduction) is also shown. Comparison of the curves for various  $\alpha$  show that the reduction of shear for a soft soil condition ( $\alpha=3$ ), when  $\lambda_T$  > 1, is less than for stiffer soils ( $\alpha \ge 6$ ), for which interaction has negligible effect. For  $\lambda_T$  < 1 the reverse situation applies, with sharp reduction of shear for  $\alpha=3$ . In all cases the coupled shear is less than the corresponding uncoupled value.

Figure 4(b) compares the dynamic amplification of eccentricity  $e_{dr}/e_r$  (arising from the effects of torsional coupling) with the corresponding recommendations of major building codes (Table 2), taking  $\zeta_1$ =0.05 and building codes (Table 2), taking  $\zeta_1$ =0.05 and  $e_r$ =0.05. The results of dynamic analysis show that greatest amplification occurs at, or that greatest amplification occurs at, or around  $\lambda_T$ =1 for rigidly based buildings (see around  $\lambda_T$ =1 for rigidly based buildings (see also Tsicnias & Hutchinson 1981, Chandler & also Tsicnias & Hutchinson 1987), whilst for buildings

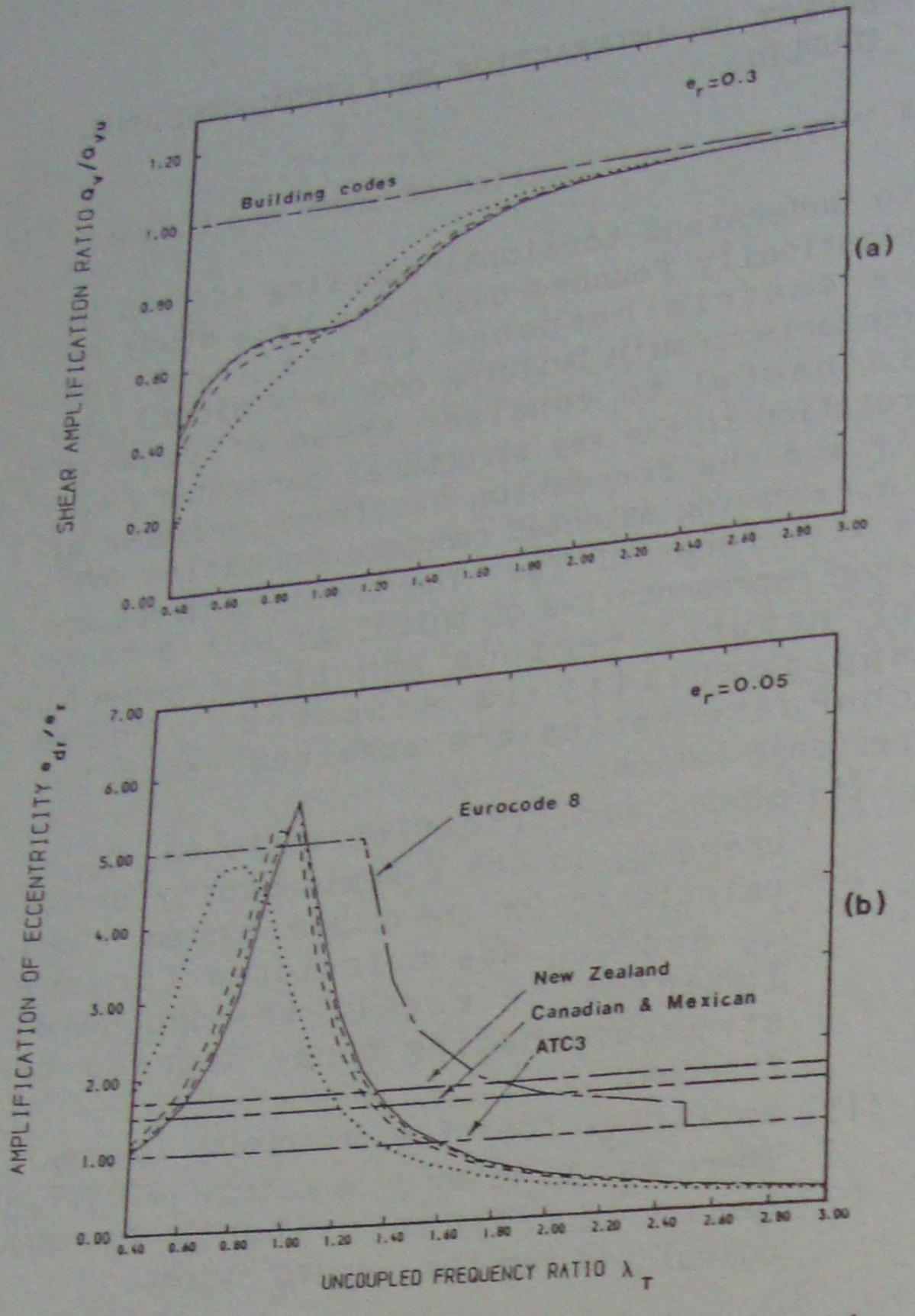


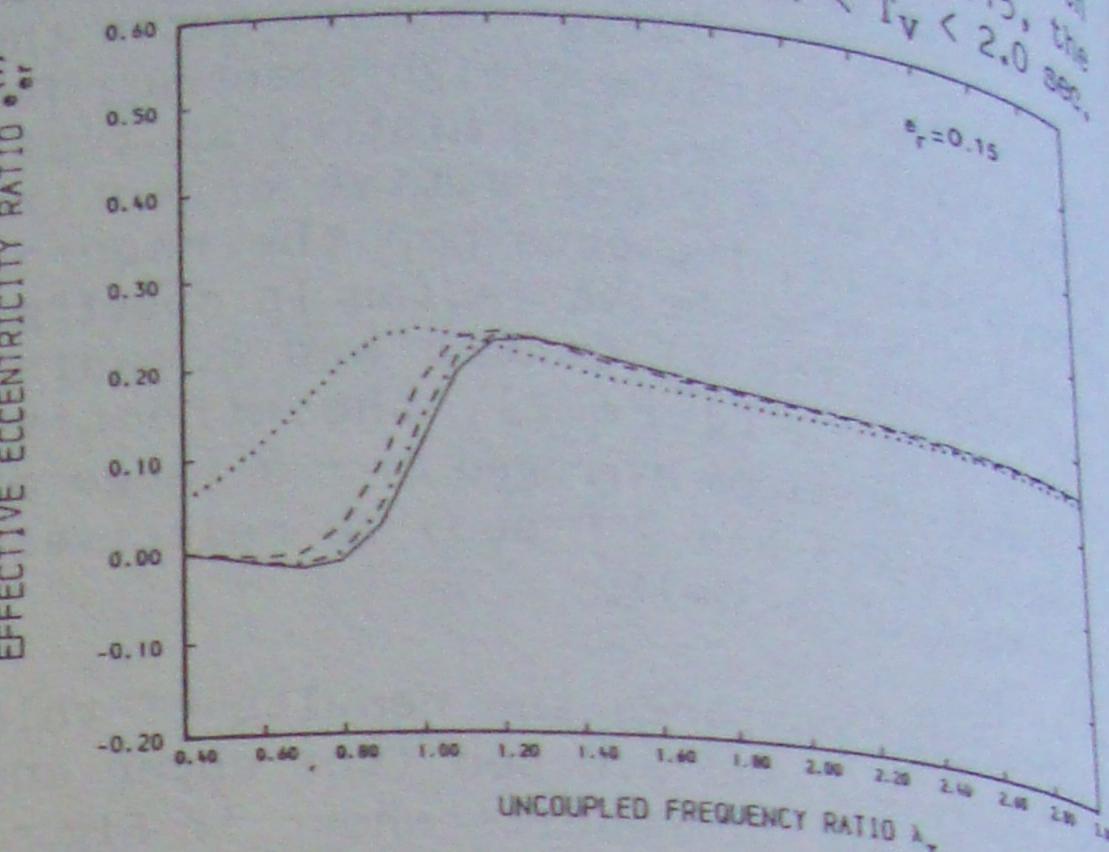
Fig. 4. Variation of the dynamic amplification of shear (a) and eccentricity (b) with  $\lambda_T$  for the flexibly based building model with  $\zeta_1$ =0.05, and comparison with the corresponding values given by building codes (---);  $\alpha$ =3 (....),  $\alpha$ =6 (---),  $\alpha$ =10 (---),  $\alpha$ = $\alpha$ =0....)

supported on flexible foundations the maximum amplification occurs when  $\lambda_T$  is somewhat less than unity (when  $\alpha=3$  for example, the peak occurs at  $\lambda_T=0.8$ ). For  $\lambda_T<0.85$ , systems with flexible foundations exhibit much higher amplification ratios than for corresponding rigidly based buildings. The significance of this result is discussed in section 4.2.

The inadequacy of most building codes in accounting for the amplification of dynamic torque due to coupling effects between lateral and torsional responses is highlighted in Figure 4(b). Within the range  $0.6 < \lambda_T < 1.4$ , representative of most actual buildings (Hart, DiJulio & Lew 1975), all codes except Eurocode Eurocode 8 gives an accurate the response. peak response but is over-conservative for  $\lambda_T$  are conservative in their estimation of

4.2 Influence of interaction on effect.

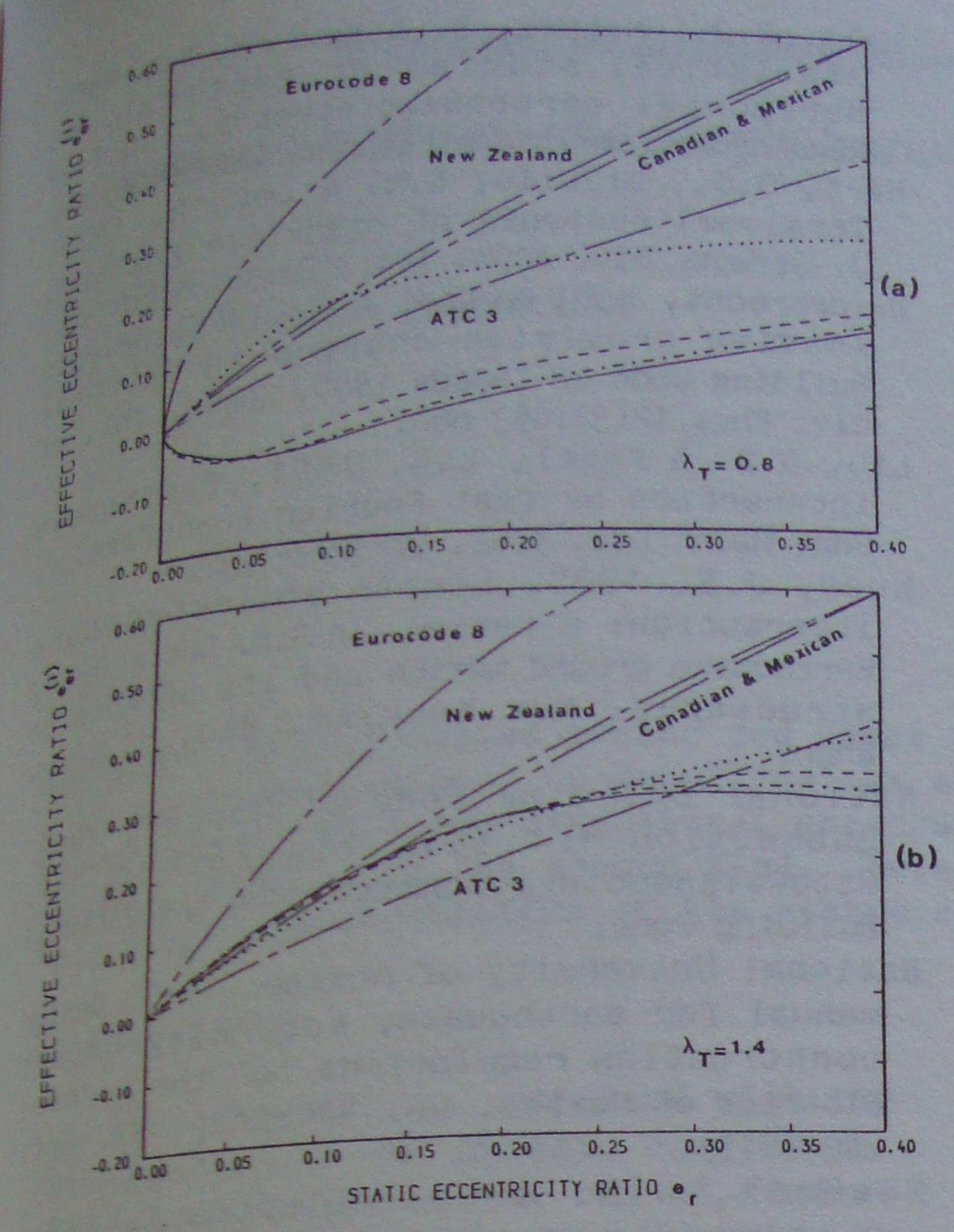
Figure 5 shows the variation of effective eccentricity ratio eer (i) (equation of effective for various α and setting er=0.15 With results being averaged over 0.1 (Ty (24))



It is shown that for  $\lambda_T < 1.2$ , interacting systems exhibit a much greater coupled response than the corresponding rigidly based structures, the effect being most pronounced for  $\alpha = 3$ . In contrast, for  $\lambda_T$  ) 1.2 interaction has little effect on response. In Figures 6(a) and 6(b) the variation of emli with er is shown, setting  $\lambda_T=0.8$  and 1.1 respectively, and comparison is made with major code provisions. In Figure 6(a), interaction induces a significant increase in effective eccentricity over the full range of er, although the building codes adequately account for this effect with the exception of buildings founded on very soft soils (a=3) when er is small or moderate. Interaction has little effect when  $\lambda_T=1.4$  (Figure 6(b)) and hence it is sufficient in this case to analyse for a rigid foundation in accounting for torsional effects. The Canadian, Mexican and New Zealand codes give a good estimate of response for er < 0.15, whilst in the same range the ATC3 and Eurocode 8 provisions are non-conservative and over-conservative, respectively. For higher values of er, all codes except ATC3 are conservative.

## 5 INCORPORATION OF INTERACTION EFFECTS IN CODE DESIGN RECOMMENDATIONS

Comparisons made in this study with current building code provisions (see Figures 4,6) indicate that inadequacies identified in several earlier studies based on rigid foundations (Tsicnias & Hutchinson 1981, Tso 1



Dempsey 1982, Chandler & Hutchinson 1987) are qualitatively unchanged when the interaction model is analysed, though detailed differences have been identified. Most codes neglect the effect of interaction in specifying equivalent lateral design forces, although the ATC3 code makes recommendations for their inclusion by means of empirical factors for reducing the calculated value of base shear. No code allowance is made for changes to the design torque provisions (Table 2) to account for soil-structure interaction.

As indicated in section 4, the results presented in Figures 4,5 and 6 show clearly that asymmetric structures supported on flexible foundations may exhibit greater coupling of lateral and torsional floor responses than the equivalent rigidly based buildings. In particular, the amplification of torsional response for  $\lambda_T < 0.85$  (that is, for buildings weak in torsion) is substantially increased by interaction effects (Figure 4(b)), and leads to increased effective eccentricity requirements in this range (Figures 5,6(a)). Hence, in these cases it is non-conservative to implement a design

for torsional coupling effects on the basis of a rigid base assumption. For  $\lambda_T > 1.2$  interaction has negligible effect on the effective eccentricity requirement (Figures 5,6(b)) and for practical design purposes, therefore, a rigid base analysis is in this case justified.

It is apparent from these results that within certain ranges of the controlling parameters er,  $\lambda_T$  and  $\alpha$ , an increased allowance for torsional coupling should be made due to the amplification of the combined lateral-torsional edge response resulting from soil-structure interaction. Further research is recommended in order to assess these effects in more detail, leading to specific recommendations for incorporation into earthquake building codes.

#### 6 CONCLUSIONS

Current torsional design recommendations in building codes are based largely on the results of simplified studies using idealised response spectra. Previous parametric studies (Tsicnias & Hutchinson 1981, Tso & Dempsey 1982, Chandler & Hutchinson 1987) have highlighted the inadequacies of these provisions in comparison with dynamic analysis, particularly in their allowance for increased lateral forces in the peripheral structural members of the building. In these studies, the ranges of the key parameters er and  $\lambda_T$  for which greatest deficiency exists in the current recommendations have been clearly identified, and various alternative provisions have been proposed (Tso & Dempsey 1982, Chandler & Hutchinson 1986). As a result of this debate, Tso (1983) made a series of recommendations to improve some of the shortcomings of the seismic torsional provisions in the National Building Code of Canada 1980. These have subsequently been adopted, together with other changes to the seismic loading provisions, in the revised National Building Code of Canada 1985 (see Heidebrecht & Tso 1985). The present study has extended previous results to incorporate an evaluation of the effects of soil-structure interaction on torsional coupling, with the following conclusions:

(1) Torsional coupling effects, as exhibited by the amplification or attenuation of the individual torsional and translational individual torsional and translational response components of the structure, are not qualitatively affected by changes in the flexibility of the foundation medium.

(2) Interaction has negligible effect on lateral-torsional coupling in structures lateral-torsional frequency ratios (λΤ) with uncoupled frequency conservative greater than 1.2, and hence conservative

design of these buildings can be achieved with the assumption of a rigid foundation. (3) For soft and medium stiff soils,

significant increases in coupled lateral-torsional response have been observed for structures having uncoupled frequency ratios (\lambda\_T) less than 1.2, that is buildings which are relatively weak in

(4) As a result of these increases, building code provisions which have been based on the response of structures supported on rigid foundations need further examination to assess the need for revision, in specific circumstances, to account for increased torsional effects resulting from

soil-structure interaction.

(5) The authors recommend that further research be carried out to assess these effects in more detail, resulting in specific design proposals in a form suitable for adoption by building codes.

#### REFERENCES

Applied Technology Council. 1978. Tentative provisions for the development of seismic regulations for buildings. Structural Engineers' Association of California, ATC 3-06. Washington: U.S. Dept. of Commerce.

Balendra, T., Tat, C.W. & Lee, S.L. 1982. Modal damping for torsionally coupled buildings on elastic foundation. Earthquake

Eng. Struct. Dyn. 10:735-756.

Chandler, A.M. 1985. Coupled torsional response of single-storey building models to earthquake loading. University of London: PhD thesis.

Chandler, A.M. 1986. Building damage in Mexico City earthquake. Nature. 320:497-501.

Chandler, A.M. & Hutchinson, G.L. 1986. Seismic design eccentricity for torsionally coupled buildings. Proc. 8th European Conf. on Earthquake Eng., Lisbon. 3(6.7):33-40.

Chandler, A.M. & Hutchinson, G.L. 1987. Evaluation of code torsional provisions by a time history approach. Earthquake Eng.

Struct. Dyn. (in press).

Chopra, A.K. & Gutierrez, J.A. 1974. Earthquake response analysis of multi-storey buildings including foundation interaction. Earthquake Eng. Struct. Dyn. 2:65-77.

Commission of the European Communities. 1984. Eurocode No. 8: Common unified rules for structures in seismic regions (draft).

Dempsey, K.M. & Tso, W.K. 1982. An alternative path to seismic torsional provisions. Soil.

Dyn. and Earthquake Eng. 1(1):3-10.

Ghaffar-Zadeh, M. & Chapel, F. 1983. Frequency-independent impedences of soil-structure systems in horizontal and rocking modes. Earthquake Eng. Struct. Dyn.

Hall, W.J., Mohraz, B. & Newmark, N.M.
Statistical studies of vertical 1976.

horizontal earthquake spectra. Nuclear
horizontal earthquake spectra. Nuclear
horizontal R.M. & Lo. 20003 norizontal norizontal Report NUREC-0003. Nucle Regulatory Commission. Report NUREC-0003.

Regulatory Communitio, R.M. & Lew, M. 1975.

Hart, G.C., DiJulio, R.M. & Lew, M. 1975.

ASCE 101(ST2):307 building. rt, G.C., Discusse of high-rise buildings.

Torsional response of high-rise buildings.

Torsional response of high-rise buildings.

J. Struct. DIV.

J. Struct. DIV.

Heidebrecht, A.C. & Tso, W.K. 1985. Seismic

Heidebrecht, Provision changes in National Nationa debrecht, A.C. loading provision changes in National loading Code of Canada 1985. Canad. Jnl Building Code of Canada 1985. Canad. Jnl. of

Civ. Eng. 12(5), Liu, S.C. & Fagel, L.W. 1971. Earthquake interaction by fast Fourier transform. J.

Mach. Div. ASCE. 97:1223-1237.

Eng. Mech. 1985. Linear soil-structure
Luco, J.E. 1985. Linear soil-structure co, J.E. Touriew. In S.K. Datta (ed.), interaction: a review. In S.K. Datta (ed.), Earthquake ground motion and its effects on Soc. of M. On structures. New York: Am. Soc. of Mech.

Engrs.

National Building Code of Canada. 1985.

Subsection 4.1.9.1, clauses 22, 23.

Associate Committee on the National Code of Canada. 1985. Ottawa: Associate Committee on the National

National University of Mexico. 1977. Design manual for earthquake, according to the construction regulations for the Federal District of Mexico. No. 406: Chap. 37, Art.

Newmark, N.M. & Rosenblueth, E. 1971. Fundamentals of earthquake engineering. Englewood Cliffs, N.J.: Prentice-Hall.

Richart, F.E., Hall, J.R. & Woods, R.D. 1970. Vibrations of soils and foundations, Englewood Cliffs, N.J.: Prentice-Hall.

Standards Association of New Zealand. 1976. Code of practice for general structural design and design loadings for buildings. NZS 4203: Part 3 (earthquake provisions), 3.4.7.2/3.

Tsai, N.C. 1974. Modal damping for soil-structure interaction. J. Eng. Mech. Div. ASCE. 100:323-341.

Tsicnias, T.G. & Hutchinson, G.L. 1981. Evaluation of code requirements for the earthquake resistant design of torsionally coupled buildings. Proc. Instn. Civ. Engrs., London. 71:821-843.

Tso, W.K. 1983. A proposal to improve the static torsional provisions for the National Building Code of Canada. Canad. Jnl. of Civ. Eng. 10(4):561-565.

Whitman, R.V. 1970. Soil-structure interaction. In R.J. Hansen (ed.), Seismic design for nuclear power plants. Cambridge, Massachusetts: MIT Press.